REFERENCES

1. Koiter, W. T., General Theorems of the Theory of Elastic-Plastic Media. IIL, Moscow, 1961.
2. Berdichevskii, V. L. and Sedov, L. I., Dynamic theory of continuously distributed dislocations. Relation to plasticity theories. PMM Vol. 31, №6, 1967.
3. Mosolov, P. P. and Miasnikov, V. P., Variational Methods in the Theory of Rigidly Viscoplastic Media. Moscow Univ. Press, 1971.
4. Sobolev, S. L., Some Applications of Functional Analysis in Mathematical Physics. Leningrad Univ. Press, 1950.
5. Eidus, D. M., On the mixed problem of elasticity theory. Dokl. Akad. Nauk SSSR, Vol. 76, №2, 1951.
6. Friedrichs, K. O., On the boundary value problems of the theory of elasticity and Korn's inequality. Ann. Math., Vol. 48, №2, 1947.
7. Sedov, L. I., Mechanics of Continuous Media. Vol. 2. "Nauka", Moscow, 1970.
8. Vainberg, M. M., Variational Methods and Method of Monotonic Operators in the Theory of Nonlinear Equations. "Nauka", Moscow, 1972.

Translated by M. D. F.

EFFECT OF INTERCONVERTIBILITY OF ELECTROMAGNETIC AND GRAVITATIONAL WAVES IN STRONG EXTERNAL ELECTROMAGNETIC FIELDS AND THE PROPAGATION OF WAVES IN THE FIELD OF A CHARGED "BLACK HOLE"

PMM Vol. 38, №6, 1974, pp. 1122-1129

G. A. ALEKSEEV and N. R. SIBGATULLIN
(Moscow)

(Received September 20, 1973)

It is shown that the behavior of an arbitrary wave propagating in the field of a nonrotating charged black hole is defined (with the use of quadratures) by four functions. Each of these functions obeys its second order equation of the wave kind. Short electromagnetic waves falling onto a black hole are reflected by its field in the form of gravitational and electromagnetic waves whose amplitude was explicitly determined. In the case of the wave carrying rays winding around the limit cycle the reflection and transmission coefficients were obtained in the form of analytic expressions.

Various physical processes taking place inside, as well as outside a collapsing star, may induce perturbations of the gravitational, electromagnetic and other fields, and lead to the appearance in the surrounding space of waves of various kinds which propagate over a distorted background and are dissipated along its inhomogeneities.

In the absence of rotation and charge in a star, the analysis of small perturbations of the gravitational fields is based on the system of Einstein equations linearized around the Schwarzschild solution. In [1, 2] this system of equations, after expansion of perturbations in spherical harmonics and Fourier transformation with respect to time, was reduced to two independent linear ordinary differential equations of second order of the form of the stationary Schrödinger equation for a particle in a potential force field. Each of these equations defines one of two possible independent perturbation kinds: "even" and "odd" (the different behavior of spherical tensor harmonics at coordinate inversion is the deciding factor in the determination of the kind of perturbation [1, 2]). Although these equations were derived with the superposition on the perturbations of the metric of specific coordinate conditions, they define, as shown in [4], the behavior of invariants of the perturbed gravitational field, which imparts to the potential barriers appearing in these equations an invariant meaning.

The system of Maxwell equations on the background of Schwarzschild solution also reduces to similar equations, which differ from the above only by the form of potential barriers appearing in these [5].

In the presence in the unperturbed solution of a strong electromagnetic field the gravitational and electromagnetic waves interact with each other, and transmutation takes place. The train of short periodic electromagnetic waves generates the accompanying train of gravitational waves. This phenomenon was first analyzed in [6] on and arbitrary background. It was shown in [7, 8] that dense stars surrounded by hot plasma may acquire a charge owing to splitting of charges by radiation pressure and the "sweeping out" of positrons nascent in vapors in strong electrostatic fields. The interaction of waves becomes particularly clearly evident in the neighborhood of black holes which may serve as "valves" by maintaining equilibrium between the relict electromagnetic and gravitational radiation in the Universe. Rotation of black holes intensifies this effect [6].

If a nonrotating star possesses an electrostatic charge, the definition of perturbations of the electromagnetic and gravitational fields must be based on the complete system of Einstein-Maxwell equations linearized around the Nordström-Reissner solution. (Small perturbations of electromagnetic field outside a charged black hole were considered in [9, 10] on the basis of the system of Maxwell equations on a "rigid" background of the Nordström-Reissner solution, without taking into account the interconvertibility of gravitational and electromagnetic waves, which materially affects their behavior in the neighborhood of a charged black hole). Here this system of equations which define the interacting gravitational and electromagnetic perturbations are reduced to four independent second order differential equations, two for each kind of perturbations (an importsnt part is played here by the coordinate conditions imposed on the perturbations of the metric, proposed by the authors in [4]). Perturbation components of the metric and of the electromagnetic field are determined in quadratures by the solutions of these equations. If the charge of a star tends to vanish, two of the derived equations convert to equations for gravitational waves on the background of the Schwarzschild solution [1, 2], while the two others become equations which are equivalent to Maxwell solutions on the same background. The short-wave asymptotics of derived equations is determined throughout including the neighborhood of the limit cycle for the wave carrying rays. These solutions far away from the point of turn coincide with those obtained in [6] for any arbitrary background. Approximation of geometric optics does not provide correct asymptotics for impact parameters of rays which are close to critical for which the isotropic and geodesic parameters wind around the limit cycle. This case is investigated below.

A similar situation in the Schwarzschild field was analyzed in [11], where analytic expressions for the wave reflection and transmission coefficients were determined, and the integral radiation stream trapped by a black hole produced by another radiation component of the dual system was calculated.

1. Derivation of fundamental equations. The external field of a charged nonrotating spherically-symmetric black hole is defined by the electro-vacuum Nordström-Reissner solution

$$ds^2 = e^{\nu}c^2dt^2 - e^{-\nu}dr^2 - r^2(dQ^2 + \sin^2\theta d\varphi^2) \tag{1.1}$$
$$F_{0r} = -E,\ E = Q/r^2,\ m = \gamma M/c^2,\ q^2 = \gamma Q^2/c^4,\ e^{\nu} = 1 - 2m/r + q^2/r^2$$

where Q is the charge and M the mass of the black hole.

1.1. Notation and formulas. Indices $\alpha, \beta, \gamma, \ldots = 0, 1$; $a, b, c, \ldots = 2, 3$; $i, j, k, \ldots = 0, 1, 2, 3$; coordinates $x^0 = ct$, $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$; $g_{\alpha\beta}$ and g_{ab} are metric tensors on coordinate surfaces (x^0, x^1) and (x^2, x^3), respectively, induced by metric g_{ij} of solution (1.1). The raising and lowering of indices $\alpha, \beta, \ldots$ and $a, b, \ldots$ is effected with the use of metrics $g_{\alpha\beta}$ and g_{ab}, respectively, ∇_α and ∇_a are operators of covariant differentiation at coordinate surfaces (x^0, x^1) and (x^2, x^3) constructed by metrics $g_{\alpha\beta}$ and g_{ab}; $\alpha_{\alpha\beta}$ and β_{ab} are Levi-Civita tensors at these surfaces. The nonzero components of Maxwell tensors for the Nordström-Reissner solution are

$$F_{\alpha\beta} = -\alpha_{\alpha\beta}E, \quad \mu = \ln r^2, \quad \mu_\alpha = \nabla_\alpha\mu, \quad \nabla_\alpha E = -\mu_\alpha E$$
$$\Box_{01} = g^{\alpha\beta}\nabla_\alpha\nabla_\beta, \quad \Box_{23} = g^{ab}\nabla_a\nabla_b = -\Delta/r^2$$

where Δ is the Laplace operator on a two-dimensional sphere of unit radius.

The perturbations of metric h_{ik} related to coordinate transformation on the sphere decompose into sets of scalars h_{00}, h_{01} and h_{11}, vectors h_{0a} and h_{1a}, and tensor h_{ab}. Scalars $h_{\alpha\beta}$ belong to perturbations of the even kind. Vectors $h_{\alpha a}$ ($h_{\alpha a}$ denotes the set of two vectors: h_{0a} and h_{1a}) and tensor h_{ab} can be represented in the form

$$h_{\alpha a} = \beta_{ab}\nabla^b h_\alpha + \nabla_a H_\alpha, \quad h_{ab} = \beta_{ac}\nabla_b\nabla^c D + \beta_{bc}\nabla_a\nabla^c D + \nabla_a\nabla_b G + g_{ab}K$$

where h_α, H_α, G, D and K are scalar functions. The terms which contain β_{ab} correspond to the odd component of perturbations, and the remaining to even ones. A similar decomposition into components of different parities can be carried out, also, for perturbations of components of the Maxwell tensor (formulas are given below).

1.2. Coordinate conditions. Coordinate conditions which can be satisfied by a particular choice of the infinitesimal coordinate transformation $x^{i'} = x^i + \xi^i(x^0, x^1, x^2, x^3)$ may be imposed on small perturbations of metric h_{ik}. Such transformation alters h_{ik} thus: $h_{ik}' = h_{ik} - \nabla_i\xi_k - \nabla_k\xi_i$. The four-vector quantity ξ_i can be decomposed into components: scalars ξ_0 and ξ_1, and even vectors $\nabla_a\xi$ and the odd vector $\beta_{ab}\nabla^b\eta$. The even component of vector ξ_i contains three arbitrary functions (ξ_0, ξ_1, ξ), and the odd one has a single one (η). It is, consequently, possible to impose on the even perturbations three conditions and on the odd ones a single condition.

Let us impose the following coordinate conditions:

$$h_\alpha^{\alpha'} \equiv h_0^{0'} + h_1^{1'} = 0, \qquad h_{ab}' = 0 \tag{1.2}$$

which for the scalar functions yields

$$\eta = D,\ \xi = G/2,\ \xi^1 = Kr/2,\ (\partial/c\partial t)\xi^0 + (\partial/\partial r)\xi^1 = h_\alpha^\alpha/2$$

where the specific form of metric (1.1) is taken into consideration. It will be seen that the ξ^i necessary for satisfying conditions (1.2) are determined by the perturbations of the metric in quadratures, hence the imposed coordinate conditions can always be satisfied.

It is assumed in what follows that quantities $h_{\alpha\beta}$, h_α and H_α are tensors with respect to transformation of coordinates on surface (x_0, x^1).

1.3. Odd perturbations. If the coordinate conditions are satisfied, the odd components of metric tensor perturbations are of the form

$$h_{\alpha\beta} = 0, \quad h_{\alpha a} = \beta_{ab}\nabla^b h_\alpha, \quad h_{ab} = 0 \tag{1.3}$$

Perturbations of the Maxwell tensor components F_{ik} and its dual tensor $\overset{*}{F}_{ik} = {}^1\!/_2\varepsilon_{iklm}F^{lm}$ are defined by

$$\begin{aligned} &\delta F^{\alpha\beta} = 0, \quad \delta F^{\alpha a} = \beta^{ab}\nabla_b F^\alpha, \quad \delta F^{ab} = \beta^{ab}\delta H \\ &\delta\overset{*}{F}{}^{\alpha\beta} = \alpha^{\alpha\beta}\delta H, \quad \delta\overset{*}{F}{}^{\alpha a} = \nabla^a(\alpha^{\alpha\beta}F_\beta - Eh^\alpha), \quad \delta\overset{*}{F}{}^{ab} = 0 \end{aligned} \tag{1.4}$$

1.3.1. Maxwell equations for perturbations. When conditions (1.2) are satisfied, the determinant g of the metric tensor remains unperturbed and the Maxwell equations for perturbations assume the form

$$(1/\sqrt{-g})\,\partial_j(\sqrt{-g}\,\delta F^{ij}) = 0, \quad (1/\sqrt{-g})\,\partial_j(\sqrt{-g}\,\delta\overset{*}{F}{}^{ij}) = 0 \tag{1.5}$$

Using the notation introduced above, these equations can be written as

$$\begin{aligned} &\nabla_\beta F^\beta = \delta H, \quad \alpha^{\alpha\beta}\nabla_\alpha F_\beta + E\mu^\alpha h_\alpha = 0 \\ &\nabla_\beta\delta H + \mu_\alpha\delta H + \Box_{23}(F_\beta - E\alpha_{\beta\gamma}h^\gamma) = 0 \end{aligned} \tag{1.6}$$

Applying operator ∇^β to the last of Eqs. (1.6) and eliminating F_β with the use of the remaining of these equations, we obtain

$$\Box_{01}(\delta H/E) + \Box_{23}(\delta H/E) = -\Delta[\alpha^{\alpha\beta}\nabla_\alpha(h_\beta/r^2)] \tag{1.7}$$

1.3.2. Perturbations of components of the energy-momentum tensor. Perturbations of the electromagnetic field (1.4) induce perturbations of the energy-momentum tensor components

$$\begin{aligned} &\delta T_{\alpha\beta} = 0, \quad \delta T_{ab} = 0 \\ &\delta T_{\alpha a} = \beta_{ab}\nabla^b[\varepsilon h_\alpha - (E/4\pi)\,\alpha_{\alpha\beta}F^\beta], \quad \varepsilon = E^2/8\pi \end{aligned} \tag{1.8}$$

1.3.3. Linearized Einstein equations. Perturbations of the Ricci tensor components induced by the perturbations of metric (1.3) are of the form

$$\begin{aligned} &\delta R_{\alpha\beta} = 0 \\ &2\delta R_{\alpha a} = -\beta_{ab}\nabla^b[\Box_{01}h_\alpha + \Box_{23}h_\alpha - \nabla_\alpha\nabla_\beta h^\beta + \mu_\alpha\nabla_\beta h^\beta - {}^1\!/_2R_* h_\alpha - \\ &\quad \nabla_\alpha(\mu_\beta h^\beta) + (2\nabla_\alpha\mu_\beta + \mu_\alpha\mu_\beta)h^\beta], \quad R_* = (\partial^2/\partial r^2)\,e^\nu/2 \\ &2\delta R_{ab} = \beta_{ac}\nabla_b\nabla^c\nabla_\alpha h^\alpha + \beta_{bc}\nabla_a\nabla^c\nabla_\alpha h^\alpha \end{aligned} \tag{1.9}$$

where R_* is the curvature scalar for metric $g_{\alpha\beta}$. On the strength of Einstein equations from (1.8) and (1.9), we have

$$\nabla_\alpha h^\alpha = 0 \tag{1.10}$$

$$\Box_{01}h_\alpha + \Box_{23}h_\alpha - \nabla_\alpha(\mu_\beta h^\beta) + (2\nabla_\alpha\mu_\beta + \mu_\alpha\mu_\beta)h^\beta - {}^1\!/_2R_* h_\alpha = (\varkappa\varepsilon/2\pi)\,\alpha_{\alpha\beta}F^\beta - 2\varkappa\varepsilon h_\alpha \tag{1.11}$$

Terms which vanish because of (1.10) are omitted in (1.11). We convolute (1.11) with μ^α, apply to both parts the Laplace operator and, using (1.6), eliminate from the right-hand part F^β. We obtain

$$\Box_{01}h + \Box_{23}h + [{}^3/_2 R_* - 5\varkappa\varepsilon]\, h = 4\varkappa\varepsilon r^2 \alpha^{\alpha\beta}\mu_\alpha \nabla_\beta (\delta H / E), \quad h = \Delta\mu_\alpha h^\alpha \tag{1.12}$$

1.3.4. The closed system of equations. Let us consider Eq. (1.7). Using (1.10) – (1.12) we express its right-hand part in terms of h by applying to (1.7) the operator $r\alpha^{\alpha\beta}\mu_\alpha\nabla_\beta$, which in coordinates (ct, r, θ, φ) reduces to $2\partial / c\partial t$ and is, consequently, permutative with all remaining differentiation operators. As the result we have

$$\Box_{01}\varphi + \Box_{23}\varphi + 4\varkappa\varepsilon\varphi + (\Delta + 2)\, h / r^3 = 0, \quad \varphi = r\alpha^{\alpha\beta}\mu_\alpha\nabla_\beta (\delta H / E) \tag{1.13}$$

with (1.12) assuming the form

$$\Box_{01}h + \Box_{23}h + [({}^3/_2)R_* - 5\varkappa\varepsilon]h = 4\varkappa\varepsilon r\varphi \tag{1.14}$$

Equations (1.13) and (1.14) constitute a closed system. Components of the electromagnetic and gravitational field perturbations are determined by solving that system together with the remaining Einstein and Maxwell equations in the form of series in spherical functions whose coefficients are computed in quadratic form. In coordinates (ct, r, θ, φ) these equations are of the form

$$\begin{gathered} Lh = -6\,(m / r^3)\, h - 4\,(q^2 / r^3)\, \varphi, \qquad L\varphi = (\Delta + 2)\, h / r^3 \\ r^* = \int e^{-\nu}\, dr, \qquad L = -\Box_{01} - \Box_{23} - 4q^2 / r^4 \end{gathered} \tag{1.15}$$

where $l \geqslant 2$, since for $l = 0, 1$ the derivation of these equations looses its meaning.

It is evidently possible to introduce new variables η_+ and η_- by formulas

$$\eta_\pm = C_\pm h + 4q^2\varphi, \quad C_\pm = 3m \pm \sqrt{9m^2 - 4q^2(\Delta + 2)} \tag{1.16}$$

such that system (1.15) is decomposed into two independent second order equations each of which contains only one unknown (the plus or minus sign are chosen to suit the unknown η_+ and η_-)

$$(\partial^2 / \partial r^{*2} - \partial^2 / c^2\partial t^2)\, \eta_\pm + (\Delta / r^2 + C_\pm / r^3 - 4q^2 / r^4)\,(1 - 2m / r + q^2/r^2)\eta_\pm = 0 \tag{1.17}$$

1.4. Even perturbations. A procedure similar to the above can be also applied to even perturbations. For coordinate conditions (1.2) we have the following nonzero components of even perturbations of the metric, and of the Maxwell tensor and its dual

$$\begin{gathered} h_{\alpha\beta} \quad (h_\alpha{}^\alpha = 0), \qquad h_{\alpha a} = \nabla_a H_\alpha \\ \delta F^{\alpha\beta} = -\alpha^{\alpha\beta}\delta E, \qquad \delta F^{\alpha a} = \nabla^a f^\alpha, \qquad \delta F^{ab} = 0 \\ \delta \overset{*}{F}{}^{\alpha\beta} = 0, \qquad \delta \overset{*}{F}{}^{\alpha a} = -\beta^{ab}\nabla_b (\alpha^{\alpha\beta} f_\beta - E H^\alpha), \qquad \delta \overset{*}{F}{}^{ab} = \beta^{ab}\delta E \end{gathered}$$

1.4.1. Maxwell equation for perturbations. For even perturbations we write Eqs. (1.5) in the form

$$\nabla_\beta f^\beta = 0, \quad \Delta f^\alpha + \alpha^{\alpha\beta}\nabla_\beta (r^2\delta E) = 0, \quad \alpha^{\alpha\beta}\nabla_\alpha f_\beta + E\mu_\alpha H^\alpha + \delta E = 0 \tag{1.18}$$

Eliminating f^α, as in the case of odd perturbations, from (1.18) we obtain

$$\Box_{01}\Psi + \Box_{23}\Psi - \Delta\,(\mu_\alpha H^\alpha / r^2) = 0, \quad \Psi = \delta E / E \tag{1.19}$$

Below we shall also use the equality

$$\Delta\alpha^{\alpha\beta}\mu_\alpha f_\beta = -\mu^\alpha\nabla_\alpha\Psi \tag{1.20}$$

which follows from (1.18).

1.4.2. Perturbations of components of the energy-momentum tensor.

$$\delta T_{\alpha\beta} = (E\delta E / 4\pi)\, g_{\alpha\beta} + \varepsilon h_{\alpha\beta} \qquad (1.21)$$
$$\delta T_{\alpha a} = \nabla_a [-(E/4\pi)\, \alpha_{\alpha\beta} f^{\beta} + \varepsilon H_{\alpha}]$$
$$\delta T_{ab} = -(E\delta E / 4\pi)\, g_{ab}$$

1.4.3. Linearized Einstein equations. Perturbations of the Ricci tensor components

$$2\delta R_{\alpha\beta} = \nabla_{\gamma} (\nabla_{\alpha} h_{\beta}{}^{\gamma} + \nabla_{\beta} h_{\alpha}{}^{\gamma} - \nabla^{\gamma} h_{\alpha\beta}) + \mu_{\gamma} (\nabla_{\alpha} h_{\beta}{}^{\gamma} + \nabla_{\beta} h_{\alpha}{}^{\gamma} - \nabla^{\gamma} h_{\alpha\beta}) + \qquad (1.22)$$
$$\square_{23} (\nabla_{\alpha} H_{\beta} + \nabla_{\beta} H_{\alpha} - h_{\alpha\beta})$$
$$2\delta R_{\alpha a} = \nabla_a [-\square_{01} H_{\alpha} + \nabla_{\alpha} (\mu_{\beta} H^{\beta}) - (2\nabla_{\alpha}\mu_{\beta} + \mu_{\alpha}\mu_{\beta}) H^{\beta} + \nabla_{\beta} h_{\alpha}{}^{\beta} + (1/2)\, R_{*} H_{\alpha}]$$
$$2\delta R_{ab} = g_{ab} [\nabla_{\alpha} (\mu_{\beta} h^{\alpha\beta}) + \mu_{\alpha}\mu_{\beta} h^{\alpha\beta} + \square_{23} (\mu_{\alpha} H^{\alpha})] + 2\nabla_a \nabla_b \nabla_{\alpha} H^{\alpha}$$

Fo even perturbations it is convenient to carry out transformations with all equations expressed in terms of coordinates (ct, r, θ, φ). By virtue of the Einstein equations, taking into account (1.20), from (1.21) and (1.22) we have

$$(\partial^2 / \partial r^{*2} - \partial^2 / c^2\partial t^2)\, H + (6m / r^3 - 8q^2 / r^4)\, e^{\nu} H + (2\Delta / r)\, [(\partial / c\partial t) h_0{}^1 + \qquad (1.23)$$
$$(\partial / \partial r^*)\, h_{00}] = -8\varkappa\varepsilon r e^{\nu} (\partial / \partial r^*)\, \Psi$$
$$(2 / r)(\partial / c\partial t)\, h_0{}^1 + (2 / r^2)(\partial / \partial r^*)(h_{00} r) - e^{\nu} H / r^2 = -4\varkappa\varepsilon e^{\nu}\Psi$$
$$(2 / r)(\partial / c\partial t)\, h_{00} - (1 / 2r)(\partial / c\partial t)\, H + (\Delta / r^2)\, [e^{\nu} (\partial / \partial r^*)\, H^0 + h_0{}^1] = 0$$
$$(2 / r)(\partial / c\partial t)\, h_0{}^1 + (1 / r^2)(\partial / \partial r^*)(rH) + \Delta h_{00} / r^2 - (\nu' / 2r)\, e^{\nu} H = 0$$
$$(\partial / \partial r) H^1 + (\partial / c\partial t)\, H^0 = 0$$
$$(\Delta = -l(l+1),\ \ l \geqslant 2,\ \ H = 2\Delta H^1 / r = \Delta\mu_{\alpha} H^{\alpha},\ \ \nu' = d\nu / dr)$$

The derivation of these equations is analogous to the derivation of equations for even perturbations of a gravitational field on the background of the Schwarzschild metric, which was presented in detail in [4].

1.4.4. The closed system of equations. We introduce new variables

$$M = rh_{00} - Hr / 2,\ \ N = \Delta h_{00} + (1 - 3m / r + 2q^2 / r^2)\, H$$

and, eliminating $h_0{}^1$ from the second and fourth of Eqs. (1.23), obtain

$$2(\partial / \partial r^*)\, M = N - 4\varkappa\varepsilon r^2 e^{\nu}\Psi \qquad (1.24)$$

It follows from Eqs. (1.23) that

$$(\partial / \partial r^*)\, N / 2 - (\partial^2 / c^2\, \partial t^2)\, M + e^{\nu}\Delta M / r^2 - (2e^{\nu} / rp(r))\, (3m / r - \qquad (1.25)$$
$$4q^2 / r^2)(N - \Delta M / r) - 2\varkappa\varepsilon r^2 e^{\nu} (\partial / \partial r^*)\, \Psi = 0$$
$$(p(r) = \Delta + 2 - 6m / r + 4q^2 / r^2)$$

Eliminating N from (1.24) and (1.25) and substituting $M = Qp(r)$, we obtain

$$(\partial^2 / \partial r^{*2} - \partial^2 / c^2\partial t^2)\, Q + [\Delta / r^2 + (6m / r - 4q^2 / r^2)\, U(r) / r^2 + \qquad (1.26)$$
$$8q^2 e^{\nu} / r^4 p(r)]\, e^{\nu} Q] + 2\varkappa\varepsilon r e^{\nu} U(r)\, \Psi = 0$$
$$U(r) = (\Delta^2 - 4 + 12m / r - 12m^2 / r^2 + 4mq^2 / r^3) / p^2(r)$$

In new variables Eq. (1.19) assumes the form

$$(\partial^2 / \partial r^{*2} - \partial^2 / c^2\partial t^2)\, \Psi + [\Delta / r^2 + 8e^{\nu} q^2 / r^4 p(r)]\, e^{\nu}\Psi + \qquad (1.27)$$
$$(4e^{\nu} / r^2)(\partial / \partial r^*)\, Q + [4e^{\nu} (3m / r - 4q^2 / r^2) / p(r) - \Delta]\, (2e^{\nu} / r^3)\, Q = 0$$

Equations (1.26) and (1.27) constitute a closed system whose solutions determine all perturbation components in the form of quadratures. This system decomposes similarly to system (1.15) into two independent equations by the introduction of new variables

$$\chi_\pm = (C_\pm - 4q^2 / r)\, Q + 2q^2\Psi \tag{1.28}$$

($C_\pm$ is defined in (1.16)). Variables $\chi_\pm$ satisfy equations

$$(\partial^2 / \partial r^{*2} - \partial^2 / c^2\partial t^2)\, \chi_\pm + [\Delta / r^2 + (c_\pm - 4q^2 / r)\,(\Delta^2 - 4 + 12m / r - 12m^2 / r^2 + 4mq^2 / r^3) / r^3 (\Delta + 2 - 6m / r + 4q^2 / r^2)^2 + 8e^\nu q^2 / r^4 \times (\Delta + 2 - 6m / r + 4q^2 / r^2)]\, e^\nu \chi_\pm = 0 \tag{1.29}$$

2. Propagation of short waves in the Nordström-Reissner field. The effect of interference of electromagnetic and gravitational waves is particularly strongly evident in the case of short waves in which the ratio of the spherical harmonic number l to frequency ω has the meaning of the impact parameter of the wave carrying ray. Retaining in the potential barriers (1.17) and (1.29) the first two terms of expansions in inverse powers of ω we obtain

$$(d^2 / dr^{*2})\, \zeta_\pm + \omega^2 [1 - e^\nu p^2 / r^2 \pm 2e^\nu qp / \omega r^3]\, \zeta_\pm = 0, \quad p = l / \omega, \quad \zeta_\pm = \eta_\pm, \chi_\pm \tag{2.1}$$

The Wentzel-Kramers-Brillouin solution of Eqs. (2.1) (principal terms of expansion of solutions in inverse powers of ω) is defined by formulas

$$\zeta_\pm = A_\pm V(r)^{-1/4} \exp[i\omega\alpha(r_0^*, r^*) \mp i\beta(r_0, r)] \tag{2.2}$$

$$V(r) = 1 - \frac{e^\nu p^2}{r^2}, \qquad \alpha(r_0^*, r^*) = \int_{r_0^*}^{r^*} \sqrt{V(r)}\, dr^*$$

$$\beta(r_0, r) = qp \int_{r_0}^{r} \frac{1}{r^3 \sqrt{V(r)}}\, dr$$

Using formulas (1.16) for defining the perturbations h and φ of the gravitational and electromagnetic fields, respectively, in terms of ζ_+ and ζ_- from (2.2) we obtain

$$i\omega p \begin{Bmatrix} 4q\varphi \\ ph \end{Bmatrix} = V(r)^{-1/4} \exp[i\omega\alpha(r_0^*, r^*)]\, \{A_+ \exp[i\beta(r_0, r)] \pm A_- \exp[-i\beta(r_0, r)]\} \tag{2.3}$$

Similar expressions for even perturbations follow from (2.2) and (1.28). These results can also be obtained by the general method used in [6].

For a fixed impact parameter p, greater than some critical p_*, the equation $V(r) = 0$ determines the radius (r_a) of maximum closeness of a ray from $r^* = +\infty$ to the black hole and the radius (r_b) of maximum distance of a ray from $r^* = -\infty$ to that hole. The asymptotics defined by (2.3) are invalid in the neighborhood of these reversal points. However for a finite distance between the roots of function $V(r)$ the wave is almost perfectly reflected by the first encountered reversal point with the change of phase to $\pi / 2$; the transmission coefficient

$$T \approx \exp[(i\omega / 2)\, \alpha(r_a^*, r_b^*)]$$

is in this case exponentially small.

The length of the intermodulation period of waves increases with increasing r, as implied by the expression for $\lambda : \beta(r, r + \lambda) = 2\pi$. Hence the effect of wave interconvertibility is absent at a great distance from the black hole, where the electromagnetic and gravitational waves propagate independently of each other.

When only an electromagnetic or gravitational wave impinges on a black hole, then $|A_+| = |A_-|$. Such wave, having undergone several complete interconvertibility events into gravitational and electromagnetic waves, after perfect reflection, reaches $+\infty$ in the form of a set of electromagnetic and gravitational waves. If initially a purely electromagnetic wave $\varphi = a_0 \exp(i\omega r^*)$ impinges on the black hole from $r^* = +\infty$, then in the reflected wave the amplitude φ is equal $|a_0 \cos 2\beta (r_a, \infty)|$, and amplitude h is $|(4q / p)\, a_0 \sin 2\beta (r_a, \infty)|$. In the general case of $|A_+| \neq |A_-|$, the waves undergo along the length of the modulation period λ only partial interconvertibility, varying, for example, in the case of electromagnetic wave impingement from the minimum value B_- to the maximum B_+, where $B_\pm = \| A_+ | \pm | A_- \| / 4q\, p\omega V^{1/4}$. We stress that in all cases of such periodic transfer of energy (from one form to another) the total wave energy remains unchanged.

For impact parameters $p < p_*$ the waves are completely trapped by the black hole, since for finite difference of the complex roots of equation $V(r) = 0$ the reflection coefficient becomes exponentially small [11].

Charged black holes absorb relicit radiation in the Universe in the form of electromagnetic and gravitational waves with respective amplitudes $a_0 \cos\beta (r_1, \infty)|$ and $|(4q / p)\, a_0 \sin \beta (r_1, \infty)|$, where r_1 is the external gravitational radius of the black hole and a_0 the amplitude of the electromagnetic wave impinging from $+\infty$.

Of particular interest is the case of $p \approx p_*$ in which for impact parameters l / ω differing for short waves critical value by a magnitude of order ω^{-1} the reflection and penetration parameters become comparable [11]. For $p = p_*$ equation $V(r) = 0$ has a multiple root which corresponds to a ray winding around the limit cycle. Using the notation $\delta = \sqrt{1 - q^2 / m^2}$, we obtain for the radii of the circular orbits of massless particles (photons) the expression $r_* = m(3 + \sqrt{(1+8\delta^2)} / 2)$. The impact parameter of rays winding around these circular orbits is

$$p_* = m\,[4\delta^2 + 10 + 4\sqrt{1+8\delta^2} + (1/2\delta^2)(\sqrt{1+8\delta^2} - 1)]^{1/2}$$

If the black hole is electrically neutral, $r_* = 3m$ and $p_* = \sqrt{27}\, m$. If its charge is equal to its mass, $r_* = 2m$ and $p_* = 4\, m$. Formally the equation $V(r) = 0$ has a multiple root and under the inner horizon of events $r_2 = m(1 - \delta)$, but the corresponding impact parameters are imaginary.

In the neighborhood of a closed ray $|r - r_*| \sim O(1/\omega)$ Eqs. (2.1) for impact parameters close to critical $|p - p_*| \sim O(1/\omega)$ reduce to the equation of a parabolic cylinder

$$\begin{gathered}(r_* / p_*)^4 (d^2 / dr^{*2})\, \zeta_\pm + \\ \omega^2 [(r - r_*)^2 (6r_*^2 - p_*^2) / r_*^4 + 2(p_* - p) / p_* \pm 2q / \omega p_* r_*]\, \zeta_\pm = 0\end{gathered} \tag{2.4}$$

Substituting the variable $\xi = \sqrt{\omega p_*}\, (6r_*^2 - p_*^2)^{1/4} (r - r_*) / r_*^2$, into (2.4), we obtain the Weber equation in its canonical form

$$\begin{gathered}(d^2 / d\xi^2)\, \zeta_\pm + (\xi^2 + a_\pm(q))\, \zeta_\pm = 0 \\ a_\pm(q) = 2\omega p_* (6r_*^2 - p_*^2)^{-1/2} (p_* - p \pm q / \omega p_* r_*)\end{gathered}$$

Using the theory of the degenerate hypergeometric function it is possible to show that the analytic continuation of the solution corresponding to the transmitted wave yields the sum of two waves: an incident and a reflected wave. The transmission coefficient T is in modulo equal to $[\exp(-\pi a) + 1]^{-1/2}$, while the modulus of the reflection coefficient

R is equal $[\exp(\pi a) + 1]^{1/2}$. Hence in the Wentzel-Kramers-Brillouin solutions (2.3) the ratio of moduli A_- and A_+ in a reflected wave changes in comparison with the incident wave thus:

$$(|A_-|/|A_+|)_R = (|A_-|/|A_+|)[\exp(\pi a_-) + 1]^{1/2}[\exp(\pi a_+) + 1]^{-1/2}$$

and in the transmitted wave thus:

$$(|A_-|/|A_+|)_T = (|A_-|/|A_+|)[\exp(-\pi a_+) + 1]^{1/2}[\exp(-\pi a_-) + 1]^{-1/2}$$

We would mention in conclusion the cosmological aspect of the obtained results. Charged black holes may be some kind of valves which control the equilibrium between the relict radiation of a black body and the hypothetical gravitational radiation in the Universe.

The authors thank L.I.Sedov for valuable remarks.

REFERENCES

1. Regge, T. and Wheeler, J.A., Stability of a Schwarzschild singularity. Phys. Rev., Vol. 108, № 4, 1957.
2. Zerilli, F.J., Gravitational field of a particle falling in a Schwarzschild geometry analyzed in tensor harmonics. Phys. Rev., Vol. 2, № 10, 1970.
3. Zerilli, F.J., Tensor harmonics in canonical form for gravitational radiation and other applications. J. Math. Phys., Vol. 11, № 7, 1970.
4. Sibgatullin, N.R. and Alekseev, G.A., Gravitational waves in the field of a collapsing star. Collection: Problems of the Theory of Gravitation and Elementary Particles. № 5, Atomizdat, Moscow, 1973.
5. Price, R.H., Nonspherical perturbations in relativistic gravitational collapse. Phys. Rev., Vol. 5, № 10, 1972.
6. Sibgatullin, N.R., Interaction of short gravitational and electromagnetic waves in arbitrary external electromagnetic fields. ZhETF, Vol. 66, № 4, 1974.
7. Levich, E.V. and Siuniaev, R.A., The heating of gas by low-frequency radiation in proximity of quasars, nucleouses of Seifert galactics and pulsars. Astron. Zh., Vol. 48, № 3, 1971.
8. Shvartsman, V.F., On the generation of relativistic particles by neutron stars in the process of accretion. Astrofizika, Vol. 6, № 2, 1971.
9. Ruffini, R., Tiomno, J. and Vishveshwara, C.V., Electromagnetic field of particle moving in spherically symmetric black hole background. Nuovo Cimento Lett., Vol. 3, 1972.
10. Ruffini, R. and Zerilli, F.J., Ultrarelativistic Electromagnetic Radiation in Static Geometries: Black Holes, ed. by C. and B.S. de-Witt's, New York, Les Houches, 1972.
11. Sibgatullin, N.R., On the effects of scattering of gravitational and electromagnetic wave packet in the gravitational field of a "black hole". Dokl. Akad. Nauk SSSR, Vol. 209, № 4, 1973.

Translated by J.J.D.